

**Exercise 22**

Find the gradient vector field of  $f$ .

$$f(s, t) = \sqrt{2s + 3t}$$

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**Solution**

Calculate the gradient and call it  $\mathbf{F}$ .

$$\begin{aligned}\mathbf{F} &= \nabla f \\ &= \left\langle \frac{\partial}{\partial s}, \frac{\partial}{\partial t} \right\rangle f \\ &= \left\langle \frac{\partial f}{\partial s}, \frac{\partial f}{\partial t} \right\rangle \\ &= \left\langle \frac{\partial}{\partial s} (\sqrt{2s + 3t}), \frac{\partial}{\partial t} (\sqrt{2s + 3t}) \right\rangle \\ &= \left\langle \frac{1}{2}(2s + 3t)^{-1/2} \cdot \frac{\partial}{\partial s} (2s + 3t), \frac{1}{2}(2s + 3t)^{-1/2} \cdot \frac{\partial}{\partial t} (2s + 3t) \right\rangle \\ &= \left\langle \frac{1}{2}(2s + 3t)^{-1/2} \cdot (2), \frac{1}{2}(2s + 3t)^{-1/2} \cdot (3) \right\rangle \\ &= \left\langle \frac{1}{\sqrt{2s + 3t}}, \frac{3}{2\sqrt{2s + 3t}} \right\rangle\end{aligned}$$